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Is Galaxy Dark Matter a Property of Spacetime?

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We describe the motion of a particle in a central field in an expanding universe. Use is made of a double expansion in $1/c$ and $1/\tau$, where c and τ are the speed of light and the Hubble time. In the lowest approximation the rotational velocity is shown to satisfy $v^4 = \frac{2}{3} GMcH_0$, where G is Newton's gravitational constant, M is the mass of the central body (galaxy), and H_0 is the Hubble constant. This formula satisfies observations of stars moving in spiral and elliptical galaxies, and is in accordance with the familiar Tully–Fisher law.

1. INTRODUCTION

The problem of motion in general relativity theory is as old as general relativity itself. Soon after the theory was published, Einstein and Grommer (1927) showed that the equations of motion in general relativity follow from the Einstein field equations rather than have to be postulated independently as in other theories. This is a consequence of the nonlinearity of the field equations and the Bianchi identities. Much work has been done since then (Einstein *et al.*, 1938; Einstein and Infeld, 1949; Fock, 1957, 1959; Infeld, 1957; Infeld and Plebanski, 1960; Infeld and Schild, 1949; Bertotte and Plebanski, 1960; Carmeli, 1964a–c, 1965a–e) and the problem of motion in the gravitational field of an isolated system seems to be well understood these days (Damour, 1983).

We here formulate the problem of motion in an expanding universe, a topic which has not been discussed so far. The problem is of considerable importance in astronomy since stars moving in spiral and elliptical galaxies show serious deviation from Newtonian gravity and as is well known, the latter follows from general relativity theory in a certain approximation (Carmeli, 1982). It follows that the Hubble expansion imposes an extra constraint

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on the motion; the usual assumptions made in deriving Newtonian gravity from general relativity are not sufficient in an expanding universe. The star is not isolated from the "flow" of matter in the universe. When this is taken into account along with Newton's gravity, the result is a motion which satisfies a different law from the one determining the planetary motion in the solar system.

2. GEODESIC EQUATION

The equation that describes the motion of a simple particle is the geodesic equation. It is a direct result of the Einstein field equations $G_{\mu\nu} = \kappa T_{\mu\nu}$ ($\kappa = 8\pi G/c^4$). The restricted Bianchi identities $\nabla_\nu G^{\mu\nu} \equiv 0$ imply the covariant conservation law $\nabla_\nu T^{\mu\nu} = 0$. When volume-integrated, the latter yields the geodesic equation. To obtain the Newtonian gravity it is sufficient to assume the approximate forms for the metric $g_{00} = 1 + 2\phi/c^2$, $g_{0k} = 0$, and $g_{kl} = -\delta_{kl}$, where $k, l = 1, 2, 3$, and ϕ is a function that is determined by the Einstein field equations. In the lowest approximation in $1/c$ one then has

$$\frac{d^2x^k}{dt^2} = -\frac{\partial\phi}{\partial x^k} \tag{1}$$

$$\nabla^2\phi = 4\pi G\rho \tag{2}$$

where ρ is the mass density. For a central body M one then has $\phi = -GM/R$ and equation (2) yields, for circular motion, the first integral

$$v^2 = GM/R \tag{3}$$

where v is the rotational velocity of the particle.

3. HUBBLE'S LAW

The Hubble law asserts that faraway galaxies recede from each other at velocities proportional to their relative distances, $\mathbf{v} = H_0\mathbf{R}$, with $\mathbf{R} = (x, y, z)$. H_0 is the universal proportionality constant (at each cosmic time). Obviously the Hubble law can be written as ($\tau = H_0^{-1}$)

$$\tau^2 v^2 - (x^2 + y^2 + z^2) = 0 \tag{4}$$

and thus, when gravity is negligible, cosmology can be formulated as a new special relativity with a new Lorentz-like transformation (Carmeli, 1995a–c, 1996a, b). Gravitation, however, does not permit global linear relations like equation (4) and the latter has to be adopted to curved space. To this end one has to modify equation (4) to the differential form and to adjust it to curved space. The generalization of equation (4) is, accordingly,

$$ds^2 = g'_{\mu\nu} dx^\mu dx^\nu = 0 \tag{5}$$

with $x^0 = \tau v$. Since the universe expands radially (it is assumed to be homogeneous and isotropic), it is convenient to use spherical coordinates $x^k = (R, \theta, \phi)$ and thus $d\theta = d\phi = 0$. We are still entitled to adopt coordinate conditions, which we choose as $g'_{0k} = 0$ and $g'_{11} = g'^{-1}_{00}$. Equation (5) reduces to

$$\frac{dR}{dv} = \tau g'_{00} \tag{6}$$

This is Hubble's law taking into account gravitation, and hence dilation and curvature. When gravity is negligible, $g'_{00} \approx 1$, thus $dR/dv = \tau$, and by integration, $R = \tau v$ or $v = H_0 R$ when the initial conditions are chosen appropriately.

4. PHASE SPACE

As is seen, the Hubble expansion causes constraints on the structure of the universe which is expressed in the phase space of distances and velocities, exactly the observables. The question arises: What field equations does the metric tensor $g'_{\mu\nu}$ satisfy? We *postulate* that $g'_{\mu\nu}$ satisfies the Einstein field equations in the phase space, $G'_{\mu\nu} = KT'_{\mu\nu}$, with $K = 8\pi k/\tau^4$ and $k = G\tau^2/c^2$. Accordingly, in cosmology one has to work in both the real space and in the phase space. Particles follow geodesics in both spaces (in both cases they are consequences of the Bianchi identities). For a spherical solution in the phase space, similarly to the situation in the real space, we have in the lowest approximation in $1/\tau$ the following: $g'_{00} = 1 + 2\psi/\tau^2$, $g'_{0k} = 0$, and $g'_{kl} = -\delta_{kl}$, with $\nabla^2\psi = 4\pi k\rho$. For a spherical solution we have $\psi = -kM/R$ and the geodesic equation yields

$$\frac{d^2x^k}{dv^2} = -\frac{\partial\psi}{\partial x^k} \tag{7}$$

with the first integral

$$\left(\frac{dR}{dv}\right)^2 = \frac{kM}{R} \tag{8}$$

for a rotational motion. Integration of equation (8) then gives

$$R = \left(\frac{3}{2}\right)^{2/3} (kM)^{1/3} v^{2/3} \tag{9}$$

Inserting this value of R in equation (3), we obtain

$$v^4 = \frac{2}{3} GMcH_0 \tag{10}$$

5. GALAXY DARK MATTER

The equation of motion (10) has a direct relevance to the problem of the existence of the galaxy dark matter. As is well known, observations show that the fourth power of the rotational velocity of stars in some galaxies is proportional to the luminosity of the galaxy (Tully–Fisher law), $v^4 \propto L$. Since the luminosity, in turn, is proportional to the mass M of the galaxy, $L \propto M$, it follows that $v^4 \propto M$, independent of the radial distance of the star from the center of the galaxy, and in violation of Newtonian gravity. Here came the idea of galaxy dark matter or, alternatively, a modification of Newton's gravity in an expanding universe.

In this paper we have seen how a careful application of general relativity theory gives an answer to the problem of the motion of stars in galaxies in an expanding universe. If Einstein's general relativity theory is valid, then it appears that the galaxy halo dark matter is a property of spacetime and not some physical material. The situation resembles the one at the beginning of the century with respect to the problem of the advance of the perihelion of the planet Mercury, which general relativity theory showed was a property of spacetime (curvature).

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